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# Tense and Conditionals

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## Abstract

In this paper I explore the possibilities for developing a formal language containing both tense and conditional operators and a model theory for such a language.[1] The criteria for success will be that we may provide formal counterparts for a wide variety of English conditionals and that the truth conditions for these formal counterparts will be appropriate for the English conditionals which they represent.

Temporal relations play an essential role in determining the truth values of many and perhaps most conditional assertions. This fact is recognized and explored by many logicians including David Lewis (1979) and John Pollock (1981), yet the attention which investigators of the logic of conditionals have given to temporal relations has not in general included an explicit consideration of the interaction of tense and conditional constructions. Two exceptions are Thomason and Gupta (1980) and van Fraassen (1980) who do develop an account of the logical and semantical properties of conditional sentences based upon the occurrence of various tenses within those sentences. This paper will include a critique of this account, particularly as it is developed by Thomason and Gupta, and a "correction" of what I take to be some of the major problems of this account. Beyond that, the paper will be an exploration of issues which have not received very much attention by logicians and philosophers of language.

The next eight sections assume that time can be represented as a linearly ordered set of points or instants. Sections 1 – 3 provide background summaries of techniques developed in tense logic and of techniques developed in conditional logic, but no attempt is made in these sections to integrate tense and conditional logic. In section 2, I also make a distinction

[1]Part of the research for this paper was performed during the summer of 1981 while the author was a participant in a project in tense logic conducted at the University of Stuttgart under the direction of Professor Christian Rohrer and with the support of the Deutsche Forschungsgemainshaft. This research was also supported by a grant from the University of Georgia. I am grateful to both institutions for their support. I also wish to thank Christian Rohrer, Franz Guenthner, Dov Gabbay, and Hans Kamp for their helpful comments and criticisms.

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between two kinds of conditionals which I call material and intentional conditionals. These two kinds of conditionals will require different analyses. A semantics for a formal language containing both tense and conditional operators is developed and criticized in sections 4 and 5, and an alternative language containing special tensed conditional operators is developed, provided with a semantics, and evaluated in sections 6 and 7. The discussion in sections 4 – 7 is restricted to intensional conditionals. In section 8, we look at the affects of tense on material conditionals and at some special problems which arise in trying to distinguish intensional from material conditionals where the future tense is concerned.

Sections 9 – 11 explore an interpretation of tense which is based on a non-linear model for time, a non-deterministic, branching time. Such a conception of time allow us to entertain the Aristotelian notion that contingent future tense sentences may lack truth values. Some of the puzzling features of such a semantics for tense are emphasized when we try to adapt this semantics to a formal language containing both tense and conditional operators. I offer a semantics employing what I call pseudo-branching time as an alternative to the branching time of Thomason and Gupta, and I argue that this semantics avoids certain objectionable metaphysical assumptions found in the Thomason-Gupta account.

# 1 Tense Logic for Linear Time

We will rely on familiar techniques of tense logic in our investigation of those special problems which arise when we mix tense and conditionality. Our initial assumptions about the nature of time will be very limited. In our first semantics for tensed language, we will represent time as a set of moments or instants of time linearly ordered by an earlier-than relation. We will not be concerned with such questions as whether time has a first or last moment, whether time is dense or continuous, etc., nor with the problems involved in expressing various answers to these questions within a formal language containing tense operators. The interested reader may refer to Burgess (1984) for a survey of tense logic, including discussion of these issues.

We will begin our examination of the logic of tense by developing a formal language within which we can hope to represent various ordinary English sentences involving tense. Actually, we will simplify our task in this section and the rest of this paper by confining our attention to sentential languages. In this way we put off for the time being any problems which may arise when we try to incorporate machinery for representing tense within a quantificational language. We construct our formal language for tense logic by adding four monadic sentence operators P, F, H and G to a language for classical sentential logic which contains infinitely many sentence letters A, B, C, etc., and the usual truth-functional operators $\neg$, $\wedge$, $\vee$, $\supset$, and $\equiv$. Using p, q, r, etc., as sentence variables, we may read Pq as 'It has been the case that q', Fq as 'It will be the case that q', Hq as 'It has always been the case that q', and Gq as 'It will always be the case that q'. (Here and elsewhere I use formal sentences autonomously to denote themselves. I believe that no confusion will result from this.)

A model for our tensed language is an ordered triple $<T,\ll,[]>$ satisfying the following conditions:

1.1 $T = \phi$.

1.2 $\ll$ is a strict, total ordering of T; i.e., $\ll$ is a relation in T which is connected in T, asymmetric, and transitive.

1.3 [] is a function which assigns to each sentence q of our formal language a subset [q] of T.

1.4 $[\neg q] = T - [q]$, $[q \wedge r] = [q] \cap [r]$, and so on for the other truth-functional connectives.

1.5 $t \epsilon [Pq]$ iff there is a $t_1$ such that $t_1 \ll t$ and $t_1 \epsilon [q]$.

1.6 $t \epsilon [Fq]$ iff there is a $t_1$ such that $t \ll t_1$ and $t_1 \epsilon [q]$.

1.7 $t \epsilon [Hq]$ iff for every $t_1$ such that $t_1 \ll t$, $t_1 \epsilon [q]$.

1.8 $t \epsilon [Gq]$ iff for every $t_1$ such that $t \ll t_1$, $t_1 \epsilon [q]$.

Intuitively, T represents the set of all moments or times, $\ll$ represents the earlier-than relation, and [q] represents the set of all times at which q is true. The conditions 1.5 – 1.8 provide truth conditions for sentences containing one of our tense operators. Another way of developing our semantics would be to interpret the sentences of our formal language as being true or false over an interval of time rather than at individual times. An interval would be a subset I of T such that for any times $t,t_1,t_2 \in T$, if $t,t_1 \epsilon I$, $t \ll t_2$, and $t_2 \ll t_1$, then $t_2 \epsilon I$. This might be more appropriate for interpreting English sentences like 'He ran a mile', since it is obvious that there is no single moment of time at which it is true that he runs a mile. An interval semantics will still allow us to interpret a sentence q as being true at an individual time t since we can say that q is true at t just in case q is true at the interval whose only member is t. For further discussion of interval semantics and its advantages, see Humberstone (1979). For present purposes, we will simplify our task by avoiding examples which might require the use of an interval semantics.

# 2 Indicative, Subjunctive, Material, and Intentional Conditionals

The prime example of a conditional in English is a sentence which contains the words 'if' and 'then'. Examples of sentences of this sort are

2.1 If Anthony's door is unlocked, then he will be back soon.

and

2.2 If Anthony had left for the weekend, then he would have locked his door.

Of course, the word ‘then’ could be omitted in either of these sentences without any change in meaning. We could also reverse the order of the antecedent (grammatically, the dependent clause) and the consequent as in

2.3 Anthony would have locked his door if he had left for the weekend.

It is also possible to omit both ‘if’ and ‘then’ in conditionals like 2.2 which contain verbs in the subjunctive mood. We do this by changing the order of the subject and verb in the antecedent of the conditional as in

2.4 Had he left for the weekend, Anthony would have locked his door.

We see that while the words ‘if’ and ‘then’ readily come to mind when we think of English conditionals, there are really a number of constructions in English which may be used to produce sentences of the sort we want to consider. The important feature of the conditional sentence semantically is the presence of an antecedent and a consequent, where the antecedent expresses some condition which somehow mitigates the sense normally expressed by the consequent.

Certain constructions signal special kinds of conditionals which have their own truth conditions. Examples are ‘might’ conditionals like

2.5 If we had invited Frank, he might have come.

and ‘even if’ conditionals like

2.6 Even if we had invited Frank, he wouldn’t have come.

Note, however, that we can delete the word ‘even’ in 2.6 without a change of meaning. This means that a conditional can have the logical and semantical properties of an ‘even if’ conditional even though it does not contain the word ‘even’. I will say nothing more about these kinds of conditionals although they have some interesting properties. There is further distinction between different kinds of conditionals, however, which I will discuss. Some pairs of conditionals seem to have exactly the same structure except that the verbs in one member of the pair are all in the indicative mood while the verbs in the other member of the pair are all in the subjunctive mood. Furthermore, it is often the case that one member of such a pair is true while the other member is false. One such pair is

2.7 If Nute didn't write this paper, then someone else did.

2.8 If Nute hadn't written this paper, then someone else would have.

2.7 is true and 2.8 is false, yet the two conditionals have the same apparent antecedent and consequent. Thus, 2.7 and 2.8 represent distinct ways in which a condition contained in the antecedent of a conditional may mitigate the sense of the consequent of the conditional. 2.7 and 2.8 represent different kinds of conditionals having different truth conditions. Investigators have for the most part associated the difference between 2.7 and 2.8 with the difference in the mood of the verbs and hence distinguished 'indicative' conditionals like 2.7 from 'subjunctive' conditionals like 2.8.

Examples like these certainly point to the existence of two different kinds of conditionals in ordinary usage, but it may be a mistake to identify this difference with the difference in the moods of the verbs. Consider, for example

2.9 If President Reagan runs for another term, he will win.

2.10 If President Reagan were to run for another term, he would win.

The inclination of the native English speaker, I believe, will be to say that these two conditionals must have the same truth value. Nor is this a peculiarity of these two specific conditionals. It is difficult and perhaps impossible to find two conditionals, one indicative and the other subjunctive, involving the same future tense antecedent and consequent, which strike us as being as clearly different in their truth conditions as are 2.7 and 2.8. The difference which investigators draw between indicative and subjunctive conditionals might not be a difference which is invariably signalled by the mood of the verbs after all. It may be true that indicative and subjunctive conditionals in the past and present tenses have different truth conditions, but distinguishing future tense conditionals on the basis of mood is unreliable.

I suggest that the truth conditions for future tense conditionals are usually very much like those for past and present tense subjunctive conditionals, while past and present tense indicative conditionals have different truth conditions. David Lewis (1973) and others have suggested that all indicative conditionals have truth conditions very similar to those of the material conditionals of classical sentential logic. This seems a likely analysis for past and present tense indicative conditionals and it is the analysis which I will adopt in this paper, with some modifications to be developed in section 8. With this in mind, I propose that we adopt a new nomenclature for these two kinds of conditionals. I suggest that we call a conditional 'material' if it has the same truth conditions as the material conditionals of classical sentential logic, i.e., if the conditional is true just in case its antecedent is false or its consequent is true. I am suggesting that most and perhaps all past and present tense indicative conditionals are material conditionals. On the other hand, I propose that we call an English conditional 'intensional' if it is not material and instead has the same truth conditions which most subjunctive conditionals have. The appropriateness of this label will become clearer in the next section.

Any classification of conditionals which is based upon the moods or tenses of the verbs occurring in the conditionals is an explicitly grammatical or syntactic classification. The distinction between material and intensional conditionals, on the other hand, is a semantic distinction. The long-standing assumption which I am questioning is that there is a simple relationship between these syntactic and semantic distinctions. Of course, there may be a regular connection between the mood and tense of the verbs in the conditional and the semantic category of the conditional even if this connection is not the one I am questioning. For example, it is tempting to think that all future tense conditionals are intensional conditionals. But I believe that this would also be an oversimplification. I will attempt a better explanation of future tense indicative conditionals in section 8.

# 3 The Logic of Intentional Conditionals

In recent years we have seen a number of proposals for interpreting intensional conditionals. A review of these proposals is beyond the scope of this paper, but the interested reader may wish to consult Nute (1984). The later sections of this paper will rely upon one or the other of two model theories for a formal language for conditionals, each of which uses the notions of a possible world and of a selection function on the sentences of the language and a set of possible worlds.

When the antecedent of an intensional conditional is false, we cannot determine the truth value of the conditional by considering the truth values of its component antecedent and consequent. The simple fact is that English conditionals are not always truth-functional, and it is those conditionals which are not truth-functional that are intended by our term 'intensional'. For example, the conditional

3.1 If Reagan were bald, he could stick his elbow in his ear.

is clearly false even though both its antecedent and its consequent are false. The corresponding material conditional, of course, is true. Robert Stalnaker (1968) suggests that we evaluate such conditionals as 3.1 by performing a kind of thought experiment in which we imagine, construct, or consider counterfactual situations in which the antecedent of the conditional is true and determine whether or not the consequent is also true in these situations. Each of these situations represents a different way the world might have been, what is often referred to as a possible world. So Stalnaker's procedure for determining the truth value of an intensional conditional involves determining whether the corresponding material conditional is true in certain possible worlds where the antecedent of the intensional conditional is true. Many other proposals have shared this basic approach. The differences in these different proposals have concerned the way in which the appropriate worlds are to be chosen.

Stalnaker's particular proposal, like many others, depends upon the idea that it makes sense to talk about the relative similarity between different worlds. For a given counterfactual antecedent q, one world in which q is true may be more similar to the actual world than is some other world in which q is true. Stalnaker proposes that for any antecedent q, if it is possible for q to be true at all then there is some possible world at which q is true which is more like the actual world than is any other possible world at which q is true. If we call a world at which q is true a 'q-world', then Stalnaker's assumption is that for every sentence q, either q is impossible or there is some unique q-world which is most similar or 'closest' to the actual world.

Our formal language is obtained by augmenting the language of classical sentential logic with a special dyadic operator $>$. We will use the subjunctive mood in reading the conditional sentences of this language, e.g., we will read '$q > r$' as 'If it were the case that q, then it would be the case that r'. Stalnaker's interpretation of such a language involves what we will call world selection function models. A world selection function model for our conditional language is an ordered triple $<W,f,[]>$ satisfying the following conditions:

3.2 W is a non-empty set.

3.3 f is a function which assigns to a sentence q and a member w of W either the empty set or a member f(q,w) of W.

3.4 [] is a function which assigns to each sentence q of our conditional language a subset [q] of W.

3.5 $[\neg q] = W - [q]$, $[q \wedge r] = [q] \cap [r]$, and so on for our other truth-functional connectives.

3.6 If f(q,w) is not empty, then $f(q,w) \epsilon [q]$.

3.7 $w \epsilon [q > r]$ iff f(q,w) is empty or f(q,w) is contained in [r].

3.8 If $w \epsilon [q]$, then $f(q,w) = \{w\}$.

3.9 If $f(q,w) = \phi$, then $f(r,w) \cap [q] = \phi$.

3.10 If $f(q,w) \epsilon [r]$ and $f(r,w) \epsilon [q]$, then $f(q,w) = f(r,w)$.

Where $<W,f,[]>$ is a world selection function model, the intended interpretation of W is as a non-empty set of possible worlds, the intended interpretation of [] is as a function which tells us for each sentence q the set of those worlds at which q is true, and the intended interpretation of f is as a function which tells us for each sentence q and world w which world at which q is true is most like w. The motivation for conditions 3.2 – 3.7 should be obvious, and the motivation for 3.8 – 3.10 only slightly less obvious. The class of world selection function models characterizes Stalnaker's favorite conditional logic **C2**. Axiomatizations of **C2** and discussions of the motivation for and adequacy of Stalnaker's semantics can be found in several places, including Stalnaker (1968). The reader should be warned that the present formulation of the Stalnaker semantics differs from Stalnaker's original formulation in certain ways. In particular, we assign the empty set as the value of f(q,w) when there is no q-world at all similar to w. Stalnaker posited an absurd world at which all sentences are true to play a similar role.

One consequence of world selection function semantics which we must take note of is Conditional Excluded Middle.

CEM: $(q > r) \vee (q > \neg r)$

If f(q,w) is empty, then clearly w [q > r] by 3.7. On the other hand, if $f(q,w) = w_1$, then $w_1 \epsilon [r]$ or $w_1 \epsilon [\neg r]$ by 3.5, and thus $w \epsilon [q > r]$ or $w \epsilon [q > \neg r]$ by 3.7. So CEM is true at every world in every world selection function model. But CEM is not universally accepted as a logical truth. In fact, more authors seem to have rejected CEM than have accepted it. Consider the following two conditionals:

3.11 If Robert had wrecked his bicycle, he would have broken his arm.

3.12 If Robert had wrecked his bicycle, he would not have broken his arm.

In most contexts where the antecedent of 3.11 and 3.12 is false, we would likely say that both 3.11 and 3.12 are false. The simple fact is that if Robert had wrecked his bicycle, he might or might not have broken his arm. Despite the evidence against CEM, we consider Stalnaker's semantics here because both Thomason and Gupta (1980) and van Fraassen (1980) use Stalnaker's semantics as the foundation for their discussions of tense and conditionals.

We can avoid CEM if we allow our selection function to pick out a class of possible worlds instead of an individual world. It seems reasonable that we should consider more than one way things might be if the counterfactual antecedent of a conditional were true. Consider, for example, a roll of a die where an ace comes up. If we consider what would have happened if an ace had not come up, we will surely consider at least five different worlds, one for each of the other five values which might have come up on that roll of the die. Contrary to Stalnaker's assumption, it would seem that there is no unique closest world in which an ace is not rolled, but rather that there are several worlds which are equally similar to the actual world. We will want to consider each of these worlds in determining the truth value of a conditional like

3.13 If an ace had not come up, Clyde would have won his wager.

We would say 3.13 is true just in case Clyde wins his wager in all of these equally close alternative worlds. Furthermore, we may consider a particular world relevant to the truth value of a particular conditional even though that world is not a closest world at which the antecedent of the conditional is true. Suppose Mack has an ancient lawn-mower which will barely cut grass. On high grass, the mower stalls. Now suppose Mack's lawn is just slightly too short for the blades of the mower to hit the grass. Is the following conditional true or false?

3.14 If Mack's grass were higher, his mower would cut it.

I believe that 3.14 is not true, even though the closest worlds in which Mack's grass is higher, i.e., those worlds in which it is just barely long enough for the blades of his mower to reach it, are worlds in which his mower cuts the grass. But we would object to 3.14 on the grounds that if the grass were any more than this bare minimum higher, then the mower would stall and would not cut the grass. In many cases, we consider worlds which are close enough to suit our purposes in evaluating conditionals without regard for whether they are the very closest worlds in which the antecedent of the conditional is true.

This approach to the analysis of intensional conditionals is captured in the formal notion of a class selection function model. A class selection function model for our formal language for conditionals is an ordered triple $<W,f,[]>$ satisfying conditions 3.2, 3.4, and the following:

3.15 f is a function which assigns to each sentence q and each w in W a subset f(q,w) of W.

3.16 f(q,w) is contained in [q].

3.17 If $f(q,w) = \phi$, then $f(r,w) \cap [q] = \phi$.

3.18 $w \epsilon [q > r]$ iff f(q,w) is contained in [r].

3.19 If $w \epsilon [q]$, then $w \epsilon f(q,w)$.

3.20 If $f(q,w) \cap [r]$ is not empty, then $f(q \wedge r,w)$ is contained in $f(q,w) \cap [r]$.

3.21 If f(q,w) is contained in [r] and f(r,w) is contained in [q], then f(q,w) = f(r,w).

This semantics characterizes the conditional logic **CV** which is axiomatized in Lewis (1973) and elsewhere. It is the underlying semantics for intensional conditionals assumed by the account of tense and conditionals to be developed in this paper.

The notion of similarity of worlds which lies behind either of the two model theories summarized in this section must remain vague. Given different purposes and interests which speakers may have on different occasions, various features of the world might be considered more important than others in deciding which worlds are more similar to the actual world than others. The intuitive interpretation of class selection function models offered in this section introduces a further cause of vagueness since it allows the consideration of worlds which are reasonably similar to the actual world even though they are not most similar. This means that we not only have to decide on a particular occasion which features of the world are most important for determining similarity, but we also have to decide how similar a world has to be for us to include it in our deliberations. (For a discussion of some of the pragmatic features involved in shaping the selection function used on a particular occasion, see Nute (1980).) Despite this variability of the selection function, it is also widely accepted that any selection function we use, no matter what are the circumstances in which it is used, must at least have certain formal characteristics. The conditions proposed above for class selection functions is one suggestion about the characteristics which any suitable selection function must have.

## 4 Tense and Intentional Conditionals: the Language CT

An obvious first step in the analysis of the combined logic of tense and conditionals is the development of a formal language **CT** which contains both conditional and tense operators. Let **CT** be the language formed by augmenting the language of classical sentential logic with a conditional operator $>$ and tense operators P, F, H, and G. **CT** is obviously the result of combining the formal language for tense defined in section 1 with the formal language for intensional conditionals defined in section 3.

A model for our language of tense and conditionals will be an ordered quintuple $<T,W,\ll,f,[]>$ satisfying the following conditions for all $t,t_1 \epsilon T$, all $w,w_1 \epsilon W$, and all sentences q and r of **CT**:

4.1 T is a non-empty set.

4.2 W is a non-empty set.

4.3 $T \cap W = \phi$.

4.4 $\ll$ is a strict total ordering for T.

4.5 f is a function which assigns to every sentence $q \in$ **CT**, time $t \in T$, and world $w \in W$ a subset f(q,t,w) of W.

4.6 [] is a function which assigns to every sentence q a subset [q] of $T \times W$.

4.7 $[\neg q] = (T \times W) - [q]$, $[q \wedge r] = [q] \cap [r]$, and so on for the rest of our truth-functional connectives.

4.8 If $w_1 \epsilon f(q,t,w)$, then $<t,w_1> \epsilon [q]$.

4.9 $<t,w> \epsilon [q > r]$ iff for every $w_1 \epsilon f(q,t,w)$, $<t,w_1> \epsilon [r]$.

4.10 $<t,w> \epsilon$ [Pq] iff there is a $t_1$ such that $t_1 \ll t$ and $<t_1,w> \epsilon$ [q].

4.11 $<t,w> \epsilon$ [Fq] iff there is a $t_1$ such that $t \ll t_1$ and $<t_1,w> \epsilon$ [q].

4.12 $<t,w> \epsilon$ [Hq] iff for every $t_1$ such that $t_1 \ll t$, $<t_1,w> \epsilon$ [q].

4.13 $<t,w> \epsilon$ [Gq] iff for every $t_1$ such that $t \ll t_1$, $<t_1,w> \epsilon$ [q].

4.14 If $<t,w> \epsilon$ [q], then $w \epsilon f(q,t,w)$.

4.15 If $f(q,t,w) = \phi$, then $f(r,t,w) \cap \{w_1:<t,w_1> \epsilon [q]\} = \phi$.

4.16 If $f(q,t,w) \cap \{w_1:<t,w_1> \epsilon [r]\}$ is not empty, then $f(q \wedge r,t,w)$ is contained in $f(q,t,w) \cap \{w_1:<t,w_1> \epsilon [r]\}$.

4.17 If $f(q,t,w)$ is contained in $\{w_1:<t,w_1> \epsilon [r]\}$ and $f(r,t,w)$ is contained in $\{w_1:<t,w_1> \epsilon [q]\}$, then $f(q,t,w) = f(r,t,w)$.

These restrictions on our models for **CT** derive from the conditions on models for tense in section 1 and from the conditions on class selection function models for intensional conditionals in section 3. The connections should be obvious.

While we have defined a formal language containing both tense and conditional operators, and while we have developed a semantics for this language, our semantics effectively segregates the two notions of tense and conditionality. Notice that in the truth conditions 4.10 – 4.13 for tense operators the world mentioned in any one of these conditions remains constant. On the other hand, the time remains constant in the truth condition 4.9 for conditionals. In the next section I will explore the expressive power of our formal language **CT** and advance certain arguments to show a need to introduce operators whose truth conditions will involve 'simultaneous' change in time and world. These operators will be used to represent genuine tensed intensional conditionals.

# 5 What CT Can't Do

A great many interesting sentences of English can be symbolized in **CT** in obvious ways. For example,

5.1 If I had received an invitation, I would be at the party.

may be symbolized as $Pq > r$, and

5.2 If I had received an invitation, I would go to the party.

may be symbolized as $Pq > Fr$. But we run into difficulty when we consider the English sentence

5.3 If I had received an invitation, I would have gone to the party.

We cannot capture the full meaning of 5.3 by symbolizing it as $Pq > Pr$, for this would allow my attendance at the party to precede my receiving an invitation. Surely the intent of 5.3 is that I would have gone to the party after I received the invitation and not before. The time at which q would have been true must be later than the time at which p would have been true for the entire sentence to be true. Thus the time of the antecedent and the time of the consequent are related to each other in the truth conditions for the sentence in some essential way. How can we capture this when our tense operators only relate the times of the antecedent and consequent to the time of utterance and not to each other?

One possible solution to the problem is to try, in effect, to shift the time of utterance of the conditional part of 5.3 to the time of either the antecedent or the consequent and then to relate that time in an appropriate way to the actual time of utterance. Two possibilities would be $P(q > Fr)$ and $P(Pq > r)$. The first of these possibilities is proposed in Thomason and Gupta (1980). If this suggestion is correct, the antecedent of the conditional is in the present tense and the consequent is represented as being in the future from the point of view of the time of the antecedent. If the second suggestion is correct, it is the consequent which is represented as being in the present tense and the antecedent is represented as being in the past from the point of view of the time of the consequent. In both cases the time at which the conditional is true is represented as being in the past from the point of view of the time of utterance of 5.3. Either of these proposals captures the proper temporal relationship between the times of the antecedent and the consequent, but I fear neither adequately captures the sense of the English sentence with which we began.

Both of the formal sentences suggested as possible symbolizations of 5.3 will be true if 5.3 is true, but the converse may not be the case. Suppose I want to go to the party very badly and that I even sit by the telephone and wait for an invitation until the party is half over. I finally decide that the call is not coming. I telephone a friend and we decide to meet at a restaurant. After calling the friend, I would not go to the party even if I were to receive a belated invitation. Suppose in fact that the phone rings as soon as I hang up from talking to my friend, and that the call is the very invitation for which I have been waiting. I certainly would not say, "I'm sorry I can't come. If I had received an invitation, I would have come." This response would sound very peculiar under the circumstances. Nevertheless, both of the sentences of **CT** which we considered as symbolizations of this English sentence would be true under these circumstances.

The problem with these proposals is that the embedded conditional need only be true at some single moment in the past in order for the entire formal sentence to be true, while 5.3 requires that the embedded conditional be true during some stretch of past time. We might try to mend the situation by using the past tense operator H in place of the operator P.

Perhaps the correct representation of 5.3 is $H(q > Fr)$. But this will not work either. To see this, let's consider a slightly different example. Suppose I received an invitation, but the invitation fell behind my desk when my wife placed the mail in its usual spot. Then I might well assert the following conditional:

5.4 If I had looked behind my desk, I would have gone to the party.

But surely it is not true that I would have gone to the party if I had looked behind my desk the day before the invitation arrived, so $H(q > Fr)$ is too strong to be a correct symbolization of 5.4. In this case my intent in uttering 5.4 is, of course, that I would have gone to the party if I had looked behind my desk at any time after the invitation fell there. Perhaps what we need to do is to introduce a new tense operator akin to H but relativized to a particular period of time, in this case the period of time beginning at the moment when the invitation fell behind my desk. Using $H^*$ for this operator, our symbolization of 5.4 will then be $H^*(q > Fr)$. One problem with this proposal is that we cannot provide truth conditions for sentences containing $H^*$ using the model theoretical devices which we have assembled so far. The period of time associated with $H^*$ will change for different antecedents. What we might do is add another function g to our models which will assign to any sentence q, time t, and world w, an interval g(q,t,w) which is open on the right and for which the right limit is t. We could then say that $H^*q$ is true at t in w just in case q is true at every time $t_1$ in w for every $t_1$ in g(q,t,w). If we do something like this, we introduce a second element of vagueness in addition to the vagueness already inherent in our selection function for interpreting the conditional operator. A problem with this approach is that we can't really allow the set of times picked for q, t, and w by g to extend all the way to the time of utterance in every case. Suppose, for example, that the party was yesterday. Then it certainly isn't true that I would have gone to the party if I had looked behind my desk this morning. If we allow g(q,t,w) to be any set of times prior to t (or perhaps some such set such that for any two times $t_1$ and $t_2$ in g(q,t,w), if $t_1 \ll t_3$ and $t_3 \ll t_2$, then $t_3$ is also in g(q,t,w)), our new operator $H^*$ looks less and less like the familiar H. Furthermore, there seems to be no need for this operator in the analysis of sentences which do not involve conditionals. It would be simpler if we could get by with only one selection function f in our models and if it were the only source of contextually dependent vagueness in our semantics. This would also allow us to avoid the extra tense operator $H^*$, although we may still need to introduce new operators which combine elements of tense and conditionality.

Another difficulty with the suggestion that we use an operator like $H^*$ in our analysis is that this does not reflect very well the grammatical structure of the English sentences which we are studying. In either $H(q > Fr)$ or $H^*(q > Fr)$ the scope of the conditional operator is smaller than the scope of the tense operator H or $H^*$. Yet when we look at an English sentence like 5.3, the scope of the conditional operator appears to be the greatest possible. Other things being equal (and it must be admitted that they often are not), we should prefer formal representations of sentences of a natural language which most closely copy the surface structure of the sentences of natural language that are the objects of our analysis. In the present case, I see no way to represent the logical structure of certain English conditionals using separate tense and conditional operators and still allow the conditional operator to have greatest scope. I believe the tense and conditional constructions are inextricably intertwined in these sentences to form tensed conditional constructions which can not be analyzed into a part which is tensed and another part which is conditional.

Similar problems arise for the suggestion that we represent our original English sentence by either H(Pq > r) or H*(Pq > r), but an additional difficulty confronts this proposal. The initial reaction to 5.3 may be that the times of both antecedent and consequent are past times, but this is not a necessary condition for the truth of 5.3. There is nothing peculiar about saying, "I am not going to the party tomorrow, but I would have gone if I had received an invitation." It is clear that this construction indicates the time of the antecedent to be past, but the time of the consequent might be past, present, or future. Both H(Pq > r) and H*(Pq > r) guarantee that the time of the antecedent is past, but neither allows for the possibility that the time of the consequent be either present or future. This makes these symbolizations doubly unattractive.

We need some sort of tense operator which will be context dependent in a way in which familiar tense operators are not. The times involved in the truth conditions containing these operators will depend not only on the times of utterance (or, perhaps, 'projected' times of utterance in the case of embedded operators), but also on the particular content of the sentences to which the operators are attached. Since the need for such tense operators arises out of a consideration of problems involved in adequately representing the semantical structure of tensed conditional sentences of English, it is reasonable to think that the needed operators themselves will be tensed conditional operators of some sort. Our next task will be to develop a formal language which contains operators of this sort and a semantics for this language.

Let's review the combinations of tense and conditionals which we can represent in **CT**. Where the times of both antecedent and consequent are only indicated as being past, present, or future with respect to the time of utterance of the sentence, we have no problem. The difficulty arises when the sentence indicates something about the relation of the time of the antecedent to the time of the consequent. Again, where the time of the antecedent is the same as the time of utterance, there is no problem and we can represent the temporal relations using our language **CT**. It is only when the time of the antecedent is either past or future with respect to the time of utterance and the time of the consequent is either past or future with respect to the time of the antecedent that more sophisticated devices are needed than those provided in **CT**.

There are four situations remaining for further analysis. In the first, the time of the antecedent is earlier than the time of utterance and the time of the consequent is at least as early as the time of the antecedent. We can call such a conditional a past-past conditional. In the second, the time of the antecedent is earlier than the time of utterance and the time of the consequent is no earlier than the time of the consequent. These conditionals we can call past-future conditionals. The other two new kinds of conditionals we will call future-past conditionals and future-future conditionals. These are the four varieties of tensed conditionals which we are unable to represent in **CT**. In the next section we will develop a new formal language and model theory which can accommodate these kinds of conditionals.

# 6 Tensed Intensional Conditionals: the Language TC

In the last section we discovered evidence that there are constructions in English which combine tense and conditionality in such a way that the logical structure of these constructions cannot be represented using combinations of distinct tense and conditional operators. In this section we will develop a new formal language which contains, in addition to all the symbols of **CT**, four new tensed conditional operators which may be used to represent the four tensed conditional constructions listed at the end of section 5. These operators are >PP>, >PF>, >FP>, and >FF>. Each of these is a dyadic tensed conditional operator, and the resulting, expanded language **TC** is not just a language of tense and conditionals but also a language of tensed conditionals. Thus we can represent in **TC** five different kinds of intensional conditionals using our five distinct conditional operators.

Our new language **TC** requires a more complex model theory than that proposed for **CT**. Models for **TC** will still be ordered quintuples $<T,W,\ll,f,[]>$, but our selection function f will have some different properties and our truth function [] will have additional restrictions resulting from the new formulation of truth conditions for conditional sentences in **TC**. Since f will now be used to interpret tensed conditionals, it will be necessary for f to pick out for a sentence q, a time t, and a world w not just a set of worlds but rather a set f(q,t,w) of ordered pairs $<t_1,w_1>$ of times and worlds satisfying certain conditions regarding similarity to t and w. Essentially, we have the following new condition for all q, t, and w:

6.1 $f(q,t,w) \epsilon [q]$.

Presumably the choice of pairs $<t_1,w_1>$ in f(q,t,w) where $t_1$ is earlier than t will depend on and affect which past-past and past-future conditionals are acceptable, the choice of pairs where $t_1$ is later than t will depend on and affect which future-past and future-future conditionals are acceptable, and the choice of pairs $<t,w_1>$ in f(q,t,w) will depend on and affect which conditionals of the familiar form q > r are acceptable. We could establish separate selection functions for each of our conditional operators, but this will not be necessary.

The truth conditions for our new kinds of conditionals should be fairly obvious:

6.2 $<t,w> \epsilon$ [q >PP> r] iff for every $t_1$ and $w_1$ such that $<t_1,w_1> \epsilon f(q,t,w)$ and $t_1 \ll t$, there is a $t_2$ such that $t_2 \ll t_1$ and $<t_2,w_1> \epsilon [r]$.

6.3 $<t,w> \epsilon$ [q >PF> r] iff for every $t_1$ and $w_1$ such that $<t_1,w_1> \epsilon f(q,t,w)$ and $t_1 \ll t$, there is a $t_2$ such that $t_1 \ll t_2$ and $<t_2,w_1> \epsilon [r]$.

6.4 $\langle t,w \rangle \epsilon$ [q >FP> r] iff for every $t_1$ and $w_1$ such that $\langle t_1,w_1 \rangle \epsilon$ f(q,t,w) and $t \ll t_1$, there is a $t_2$ such that $t_2 \ll t_1$ and $\langle t_2,w_1 \rangle \epsilon$ [r].

6.5 $\langle t,w \rangle \epsilon$ [q >FF> r] iff for every $t_1$ and $w_1$ such that $\langle t_1,w_1 \rangle \epsilon$ f(q,t,w) and $t \ll t_1$, there is a $t_2$ such that $t_1 \ll t_2$ and $\langle t_2,w_1 \rangle \epsilon$ [r].

Of course, we must also amend our truth condition for untensed conditionals:

6.6 $\langle t,w \rangle \epsilon$ [q > r] iff for every $w_1$ such that $\langle t,w_1 \rangle \epsilon$ f(q,t,w), $\langle t,w_1 \rangle \epsilon$ [r].

While it is certainly possible to introduce tensed conditional operators having the interpretations suggested here, it might be the case that there are no constructions in English or any other natural language which correspond to each of these operators. In fact, there are English intensional conditionals corresponding to each of our tensed conditional operators. We have already seen that a sentence like 'If I had received an invitation, I would have gone to the party' is a past-future conditional. An example of a past-past conditional is 'If I had been admitted to the party, I would have had to have received an invitation'. 'Were I to be invited, I would go to the party' is a future-future conditional and 'Were I to be admitted to the party, I would have to have received an invitation' is a future-past conditional. The only past-past and future-past conditionals which I can suggest in the subjunctive mood involve the rather awkward phrases 'would have had to have' and 'would have to have'. Both past-past and future-past conditionals are varieties of back-tracking conditionals. (For discussions of these, see Lewis (1979) and Pollock (1981).) True back-tracking intensional conditionals are relatively rare, which may explain the fact that past-past intensional conditionals are not provided with simpler forms of expression in English. Since we rarely have an occasion in which it would be appropriate to assert such a conditional, there is no great practical need to evolve more efficient constructions for such conditionals. Of course, all the truth conditions for conditionals which have been offered above are for intensional conditionals. I shall have something more to say about tensed material conditionals later.

# 7 What TC Can Do

A major advantage which the analysis of the previous section enjoys over one which employs relativized versions of the familiar tense operators H and G is that only one selection function f appears in our models. Recall that if we were to represent a past-future conditional as $H^*(q > Fr)$ where $H^*$ is a relativized version of H, we would have to add a new item to our models, a selection function which would serve as the basis for interpreting the new operator $H^*$. We would have to add a dual operator $G^*$ to our formal language to represent future-past and future-future conditionals, and we would have to add a selection function to our models to interpret this operator as well. Adding either relativized tense operators or tensed conditional operators to our formal language makes our language more complicated, but adding tensed conditional operators rather than relativized tense operators results in considerably less complication for our model theory. Furthermore, the very grammatical structure of the English sentences we are considering indicates that these sentences are conditionals and that the conditional constructions in these sentences have greatest scope.

Despite the greater complexity of the corresponding model theory, there is a reason why we might prefer to use H* and G* rather than tensed conditional operators to represent the kinds of English conditionals we have been discussing. Consider the case of a tennis player, let's call him Franz, who suffers a fall during the opening round at Wimbledon. Fortunately for Franz, he suffers no serious injury and ultimately competes in the finals of the tournament. Later we might assert:

7.1 If Franz had broken his leg, he wouldn't have played in the finals.

After the tournament, Franz develops some soreness in his knees and consults a physician. The physician orders x-rays of his knees and examines them in the presence of Franz's coach. The coach asks the doctor if there is anything wrong with the leg Franz broke. To this the doctor replies, "Franz never broke his l eg." The doctor goes on to assert:

7.2 If Franz had broken his leg, there would be evidence of the break in the x-rays.

Here we have two tensed conditionals involving the same antecedent condition, 'Franz breaks his leg'. These two sentences present a problem since the range of times which may be considered in evaluating 7.1 is usually going to be far smaller than the range of times which may be considered in evaluating 7.2. It is obvious that for a given time t and world w our selection function f can pick out only one set f(q,t,w) of times and worlds at which Franz broke his leg, but we want to pick out quite different sets of pairs of times and worlds for 7.1 and 7.2. Use of the operator H* provides one solution to this problem. While the selection function associated with a conditional operator takes only the antecedent of the conditional as argument, the selection function which we would use to interpret H* in $H^*(q > Fr)$ would take $q > Fr$ as argument and hence, indirectly, both q and r. This would allow us to use different times in interpreting the two English conditionals. While I would prefer not to accept this proposal so long as there is no demonstrated need for H* in contexts which do not involve conditionals, we must recognize its advantages.

Dov Gabbay (1972) has suggested another approach which may help us explain the tennis player examples. For reasons which do not really involve considerations of tense at all, Gabbay proposes that the set of worlds which we consider in evaluating a conditional is always a function of both the antecedent of the conditional and the consequent of the conditional. If we follow Gabbay, then f becomes a function which assigns to sentences q and r, time t, and world w a set f(q,r,t,w) of pairs $<t_1,w_1>$ of times and of worlds similar to w such that q is true at $t_1$ in $w_1$. By making f a function of both antecedent and consequent, we are clearly able to distinguish between the truth conditions for the two conditionals concerning the tennis player since these two conditionals have different consequents. The difficulty with Gabbay's proposal is that it would force upon us an extremely weak logic for conditionals, a logic so weak that we could not even count among its theorems such theses as

7.3 $((q > r) \wedge (q > s)) \supset (q > (r \wedge s))$

(For a further discussion of Gabbay's semantics, see section 3.4 of Nute (1980a).) While Gabbay's approach would allow us to solve the immediate problem, I for one am not willing to pay the price of the very weak conditional logic which goes with it.

I think that a proper solution to our tennis player example lies not in a revision of our formal language and its semantics but rather in a careful consideration of the pragmatics of conditionals. It would be reasonable to say of the tennis player, "If he had broken his leg, he would not have played in the finals," and it would also be appropriate to say of the tennis player, "If he had broken his leg, there would be evidence of the break in his x-rays." But it would not be appropriate to utter both of these sentences on the same occasion. Contrary to what Gabbay suggests, we do not need to provide different truth conditions for these two sentences since both would not be uttered in the same context. What we need is an account of the pragmatic principles which prevent the utterance of both sentences on the same occasion. But we need more than this. Each sentence is true when uttered in appropriate circumstances and given that certain conditions hold. Given the clear meanings of the two sentences on different occasions, how can we express exactly these same two meanings on a single occasion?

As was mentioned earlier, the selection function we use to interpret conditionals on one occasion may not be the same function we use on another occasion. Furthermore, the particular function we use on a particular occasion is never fully defined. It could even be said that there really is no function which is being used on a particular occasion. Instead there is at best a partial function which becomes defined for additional arguments as a conversation progresses. It is indeterminate which times and worlds will be picked out for the antecedent 'Franz breaks his leg' until a sentence with this antecedent is actually used in a conversation. Once such a sentence is used and accepted, the speaker and the hearer have tacitly arrived at an understanding about the value of the selection function for this antecedent, an understanding which will make the sentence which has been uttered and accepted turn out to be true. These shared restrictions on the interpretation of conditionals comprise a component of what David Lewis (1979a) has called conversational score. For a further discussion of the role which conversational score plays in the interpretation of conditionals, see Nute (1980). A consequence of this view of the pragmatics of conditionals is that the selection functions occurring in our models must represent semantic ideals which we only approach in actual speech.

In the tennis player example, the value of our selection function for the antecedent 'Franz breaks his leg' will be determined by whichever of our two English conditionals occurs first in the conversation. Thus the consequent of the conditional does affect the selection of worlds to be considered in evaluating a conditional, but in a far more subtle way than Gabbay suggests. If the consequent were itself an argument for the selection function, it would not seem abnormal to assert both sentences in whichever order we wished on a single occasion and without further restriction. But this would be abnormal, for Franz might very well have played in the finals of Wimbledon if he had broken his leg several years before the tournament. The consequent does not serve as an argument for the selection function; rather, it helps to determine what the selection function itself may be. Whichever of the sentences is accepted first, it then becomes necessary to modify the antecedent of the other before it can be asserted on the same occasion. Thus we might say, "If Franz had broken his leg, the mend would show on an x-ray; and if he had broken his leg recently, he would not have played in the finals." We might also say, "If Franz had broken his leg, he would not have played in the finals; and if he had ever broken his leg, the mend would show on an x-ray." In each case, the antecedent is modified in the second sentence by the insertion of a qualifying temporal adverb like 'recently' or 'ever'. The interpretation for the unqualified antecedent is different in the two cases even though exactly the same English sentence serves as antecedent in the two examples. Once an interpretation is tacitly accepted for the unqualified antecedent, the antecedent of the other conditional must be modified so as to expand or restrict the set of times selected for the unqualified antecedent to produce the set of times appropriate to the qualified antecedent. This account may not be as simple as an account built on operators like $H^*$, or as an account like Gabbay's which makes the selection function take both antecedents and consequents as arguments, but it provides a better description of what occurs in actual discourse.

Another possibility would be to eschew a formal language of tenses and conditionals altogether. We could then attempt to provide a formal semantics directly for the particular English constructions in which we are interested. This is the approach of the Montague grammarians and there is much to be said for it. It seems much simpler to go directly from natural language to models for that language without the mediation of a formal language. But the present approach has several advantages. First, it allows us to axiomatize the logic of the regimented constructions which we use to represent the constructions of the natural language if we choose, although such axiomatization is not a goal of the present paper. Second, the simplicity of the formal language makes it easier in many instances to see the consequences of various decisions concerning our formal semantics and to see where to look in the natural language for difficult cases to test our semantics. Third, consideration of the regimented, formal language may result in a reform of ordinary usage. This third possibility may seem to be much less of a benefit to the linguist than it does to the philosopher. The philosopher is attempting in many cases to clarify the concepts underlying a particular linguistic usage and may decide that these concepts are confused and require certain refinement or correction. Thus the philosopher's analysis of language may ultimately result in the formation of new linguistic intuitions as well as a better understanding of the linguistic intuitions already shared by speakers of the language under study. Presumably the linguist, or at least the descriptive linguist, is never interested in changing usage in any way.

Our formal language **TC** allows us to represent true tensed conditionals of various sorts, and our formal semantics for **TC** allows us to interpret these conditionals. But both the language and the semantics suffer from various limitations which we have noted. First, we may require an interval semantics if we are to provide an adequate analysis of certain kinds of sentences involving both tense and conditionals. Second, there may be additional tensed conditional constructions which we can not represent even among those which do not require an interval semantics for their interpretation. Third, we can not explain puzzles like that of the tennis player example without augmenting our formal semantics with a fairly detailed pragmatics for conditionals. None of these limitations will be explored in greater detail in this paper. Nevertheless, I see none of these issues as a source of insurmountable difficulties for the account which has been provided. Rather these issues show ways in which the present account must be expanded before we can have a complete account of tensed conditional constructions in English or any other natural language.

# 8 Tense and Indicative Conditionals

I have proposed that we adopt new tensed conditional operators if we are to provide a formal language capable of representing the logic of intensional conditionals adequately. Throughout the discussion so far, I have assumed that all English subjunctive conditionals are examples of intensional conditionals, but I have also suggested early in this paper that English indicative conditionals may also be used intensionally. Now it is time that we look at indicative conditionals more closely and try to determine their logical and semantical properties more precisely. The first question I will consider in this section is whether there is a need for tensed material conditional operators parallelling the need for tensed intensional conditional operators. Next I will show why I believe that certain English indicative conditionals are used intensionally and what there is about the circumstances of such use which makes this practice reasonable.

Thomason and Gupta (1980) suggest that the distinction I have made between intensional and material conditionals is really a difference in the scope of the tense and conditional operators in the sentences affected. Look at the following two conditionals taken from Thomason and Gupta (1980) and originally due to Ernest Adams:

8.1 If Oswald didn't shoot Kennedy then Kennedy is alive today.

8.2 If Oswald hadn't shot Kennedy then Kennedy would be alive today.

Thomason and Gupta propose that 8.1 is of the form $Pq > r$ while 8.2 is of the form $P(q > r)$, where the conditional operator $>$ is provided with Stalnaker's semantics and r is the eternal sentence 'Kennedy is alive today'. Consider also the following two sentences taken from Thomason and Gupta (1980) with slight modification:

8.3 If Max missed the train then he took the bus.

8.4 If Max had missed the train then he would have taken the bus.

According to Thomason and Gupta, these conditionals are respectively of the forms $Pq > Pr$ and $P(q > Fr)$. The treatment of these four examples is consistent, the difference being that in the first pair r is taken to be an eternal sentence while in the second pair r represents an ordinary atomic sentence which is true at some times and false at others. I have already offered a critique of this kind of account for 8.2 and 8.4. Now let's consider whether this is an adequate account of conditionals in the indicative mood like 8.1 and 8.3.

Given the interpretation of most indicative conditionals as material conditionals which I am adopting in this paper, I would of course not use the conditional operator $>$ in symbolizing 8.1 and 8.3. Instead, I would use the truth-functional $\supset$ and symbolize these conditionals respectively as $Pq \supset r$ and $Pq \supset Pr$. I would agree with Thomason and Gupta that in the sentence 8.3 no relation between the times of the antecedent and the consequent is guaranteed, although such a relation is guaranteed by sentence 8.4. It is just as reasonable to assert 8.3 in a case in which we wish to claim that Max's taking the bus would explain his missing the train as it is in a case in which we wish to assert that Max's taking the bus would be a result of his missing the train.

These examples suggest that we needn't worry about the relation of the time of the antecedent to the time of the consequent in the case of material (indicative) conditionals. This being the case, we would not have the reason for inventing special tensed material conditional operators which motivated the creation of our tensed intensional conditional operators. But this is not the case. Although it may require the use of temporal adverbs to accomplish the task, we can certainly construct material conditionals which guarantee appropriate relations between their antecedents and their consequents. An example of such a conditional is:

8.5 If Max missed the train, he subsequently took the bus.

Here it is clear that the bus-taking follows the train-missing. I think that Thomason and Gupta would symbolize 8.5 as $P(q > Fr)$, i.e., in the same way that they suggest that 8.4 be symbolized. At least, this symbolization would seem to be consistent with their symbolizations of other examples. I will not press this suggestion with uncharitable vigor, however, since Thomason and Gupta do not in fact consider the conditional 8.5 and since I myself find the suggestion that 8.4 and 8.5 be symbolized in the same way very unattractive. An obvious way to avoid this consequence is to replace $>$ in the proposed symbolization of 8.5 with , thereby symbolizing 8.5 as $P(q \supset Fr)$. This would certainly indicate a difference in 8.4 and 8.5, but I still think that we don't have 8.5 right.

Let's look at a modification of an earlier example:

8.6 If Jane received an invitation then she subsequently went to the party.

We have difficulties if we represent this conditional as being of the form $P(q \supset Fr)$. If this were a correct symbolization, then so also would be $P(\neg q \vee Fr)$. Now suppose Jane in fact did receive an invitation on Tuesday but did not attend the party on Saturday. In this case we should say that 8.6 is false. Still it is true that $\neg q \vee Fr$ was true on Monday, so $P(q \supset Fr)$ is true now. This cannot be a correct symbolization of 8.6.

A more promising candidate for the logical form of 8.6 is $\neg P(q \wedge \neg Fr)$. In fact, this is almost correct. The only problem I can see with this suggestion is that it would allow for the possibility that Jane received an invitation yesterday and will go to the party tomorrow. The clear indication of 8.6, on the other hand, seems to me to be that Jane went to the party, not that she is going to the party. This possibility, that the time of the consequent is after the time of the utterance, does not appear to be open in the case of 8.6 as it is in the case of the intensional counterpart of 8.6, 'If Jane had received an invitation, she would have gone to the party'. To capture this additional element of 8.6, I suggest the symbolization $\neg P(q \wedge \neg Fr) \wedge (Pq \supset Pr)$. The second part of this symbolization is essentially the same as that proposed for 8.1 and 8.3, taking into account the fact that the antecedent of 8.1 is supposed to be an eternal sentence. The difference between a conditional like 8.6 and one like 8.4 is due to the occurrence of the temporal adverb 'subsequently'. It is the presence of this adverb which forces us to append the first conjunct in our symbolization of 8.6. I believe that 8.5 and 8.6 have exactly the same logical form. The reason I changed examples in the discussion is that the antecedent in 8.5 might well indicate a particular train leaving at a particular time. Since there would then be one and only one time at which Max could have missed the train, the possibility of there being some time at which he either did not miss the train or did take the bus even though 8.5 was false would not arise. But this peculiarity of 8.5 is due to the fact that the train left at a specific time rather than to the tense or the conditionality of the sentence.

A consideration of examples can hardly show that there is no need to augment our formal language with special tensed material conditional operators, for no matter how many examples we find which require no special operators there may remain unexamined conditionals which require such treatment. Nevertheless, I have been unable to discover any such examples. I therefore venture to propose that the language **TC**, and indeed the language **CT**, is adequate for the representation of all material conditionals whatever their tense structure or temporal adverbs may be. It is well worth noting, though, that the logical form of such English conditionals may be more complex than the account included in a typical treatment of classical sentential logic would indicate. Even without the complexities associated with intensional conditionals, the combination of tense with conditionality is no trivial matter.

All of the examples considered in this section have concerned antecedents in the past tense. A new problem arises when we consider future tense conditionals. The problem is that in English we often do not distinguish between future tense indicative conditionals and future tense subjunctive conditionals. Consider the following examples.

8.7 If Joe strikes this match, it will light.

8.8 If Joe were to strike this match, it would light.

Under what conditions would we assert one rather than the other of these two conditionals? We would be more likely to assert 8.8, I think, if we believed that it is unlikely that Joe will strike the match, and we would be more likely to assert 8.7 if we believed Joe might strike the match or if we were trying to persuade Joe to strike the match. But is there a difference in the truth conditions for the two sentences?

There is a temptation to say that 8.7 and 8.8 have exactly the same truth conditions, and that both are intensional conditionals. The cause of this temptation is that in deciding whether to accept 8.7 we have no option but to perform the very same sort of thought experiment which we would perform in evaluating 8.8. That is, we would imagine likely situations in which Joe strikes the match and consider whether or not the match lights in all of those situations. This is quite different from the position we find ourselves in with regard to

8.9 If Joe struck the match, it lit.

Here we can investigate what actually happened to determine whether 8.9 is true or false. Since the future is not open to investigation in the same way the past is, we cannot use this method for evaluating 8.7. With no alternative, we form our opinion about the truth of 8.7 in much the same way we form our opinion about 8.8. We might say that our epistemological situation with regard to 8.7 is exactly the same as our epistemological situation with regard to 8.8, while our epistemological situation with regard to pairs of past or present tense indicative and subjunctive conditionals is quite different. This explains why we do not distinguish as carefully between indicative and subjunctive conditionals in the future tense.

While the epistemological distinction between indicative and subjunctive conditionals in the past and present tenses collapses for indicative and subjunctive conditionals in the future tense, this does not mean that the difference in truth conditions also collapses. Just because we cannot now employ different methods in estimating the truth values of indicative and subjunctive future tense conditionals does not mean that these conditionals do not in fact have different truth conditions. To better determine the facts in this matter, let's consider the logic of future tense indicative conditionals and see if it differs from the logic of future tense subjunctive conditionals. A variety of logical principles which are acceptable for material conditionals are not acceptable for intensional conditionals. Among these are left monotonicity, transitivity, and contraposition. Let's consider the principle of left monotonicity as it applies to 8.7. Consider the conditional

8.10 If Joe dips the match in water and strikes it, it will light.

Not only does it seem plausible that someone would affirm 8.7 while denying 8.10, but it even seems likely. This suggests that 8.7 is being used as an intensional rather than as a material conditional. Yet it is also plausible that someone would insist that 8.10 is true because 8.7 is true, and conclude from this that Joe will not both dip the match in water and strike it. Inelegant though it may be, the honest conclusion to be drawn is that English indicative conditionals in the future tense may be used either materially or intensionally, and their intensional use is motivated by the fact that we cannot maintain the same epistemological distinction between indicative and subjunctive conditionals in the future tense that we maintain for past and present tense conditionals.

To summarize briefly, I am suggesting that all English subjunctive conditionals are probably intensional (I can find no persuasive counterexamples), that all past and present tense English indicative conditionals are probably material (again, I can find no persuasive counterexamples), and that future tense English indicative conditionals may be used either materially or intensionally. I further suggest that we need special tensed conditional operators for symbolizing English subjunctive conditionals (and other intensional conditionals), but that the resources of familiar tense logic are sufficient for representing the logical form of past and present tense English indicative conditionals (and other material conditionals).

# 9 Branching Time and Settledness

Our discussions so far have assumed that time is linear and that the earlier-than relation is a strict ordering of the set of times. An alternative account has it that the set of times together with the earlier-than relation form a tree structure with branching toward the future. Such an account is developed in Thomason (1970) and is employed in the investigation of tense and conditionals in Thomason and Gupta (1980). The position of Thomason (1970) is that contingent future tense sentences are often neither true nor false. Thomason includes a 'settledness' operator in his formal language for tense logic and uses branching time together with van Fraassen's method of supervaluations to provide an analysis of tense which admits of truth value gaps for future tense sentences. When this theory is augmented with an account of conditionals, some interesting problems arise to which Thomason and Gupta provide no adequate solution.

The formal language for which Thomason and Gupta provide a model theory is the language of classical sentential logic augmented by three tense operators P, F and L, and a conditional operator $>$. There appears to be no reason why the analysis could not be extended to a language containing the tense operators H and G, but these devices are not included in order to keep the discussion relatively simple. We may read Lq as 'It is settled that q'. Let's call this new formal language **TGL**. Thomason and Gupta actually provide two different model theories for **TGL**, but I will only discuss the first and simpler of these two semantics. The portion of the semantics which relates immediately to the analysis of conditionals is adapted from Stalnaker's semantics and hence validates the suspicious principle Conditional Excluded Middle, the CEM of section 3. Thomason and Gupta prefer their more complicated second model theory for **TGL** because it seems best equipped to preserve the thesis CEM together with certain other theses to which they are committed. Since I reject CEM in any case, I believe that the added complications of their second model theory are unnecessary.

A Thomason-Gupta (TG-)model for **TGL** is an ordered quadruple $<T,\leq,s,[]>$ such that

9.1 T is a non-empty set.

9.2 $\leq$ is a transitive relation on T such that if $t_1 \leq t$ and $t_2 \leq t$, then $t_1 \leq t_2$ or $t_1 = t_2$ or $t_2 \leq t_1$.

9.3 Ht is the set of all subsets h of T such that t  h, $\leq$ strictly orders h, and there is no subset $h_1$ of T having these two properties which properly contains h. In other words, Ht is the set of maximal chains with respect to $\leq$ which contain t.

9.4 [] is a function which assigns to each sentence q a set of pairs $<t,h>$ where h is a member of Ht.

9.5 $[\neg q] = \{<t,h>: h \in Ht\} - [q]$, and so on for the other truth-functional connectives.

9.6 $<t,h> \in [Pq]$ iff $h \in Ht$ and there is a $t_1$ in h such that $t_1 \leq t$ and $<t_1,h> \in [q]$.

9.7 $<t,h> \in [Fq]$ iff $h \in Ht$ and there is a $t_1$ in h such that $t \leq t_1$ and $<t_1,h) \in [q]$.

9.8 s is a function which assigns to each sentence q, time t, and member h of Ht either $\phi$ or a pair $<t_1,h_1>$ such that $h_1 \in Ht_1$.

9.9 $<t,h> \in [q > r]$ iff $h \in Ht$ and either $s(q,t,h) = \phi$ or $s(q,t,h) \in [r]$.

9.10 $<t,h> \in [Lq]$ iff $h \in Ht$ and for all $h_1$ in Ht, $<t,h_1> \in [q]$.

T represents the set of times and $\leq$ is an earlier-than relation on T. But $\leq$ is quite different from the earlier-than relation of our earlier models. In a TG-model, distinct times might not be related by $\leq$ at all. The relation $\leq$ imposes on T a tree-structure with branching toward the future. Ht represents all those temporal branches which go trough a particular time t. It should be noted that any two members of Ht will have the same members prior to t but different members subsequent to t. The members of Ht may be called histories which pass through t. The sentences of **TGL** are interpreted as being true or false at a time in a history, and the function [] tells us for each sentence the pairs of times and histories at which that sentence is true. The interpretation of the truth-functional and familiar tense operators are the standard ones. Additional restrictions on the function s will be modelled after the restrictions for Stalnaker's semantics listed in section 3. Thomason and Gupta also include a second, equivalence relation on their models. This 'co-presence' relation $\approx$ satisfies the conditions that (i) if $t \approx t_1$ then not $t \leq t_1$, and (ii) if $t \approx t_1$ and $t_3 \approx t_2$ and $t \leq t_2$ then not $t_2 \leq t$. Although this co-presence relation plays a role in their second, more complicated model theory, it is not mentioned in any of the truth conditions for their first model theory and I have therefore omitted it for the sake of simplicity.

The feature of this theory which attracts our interest is the analysis of the operator L. The intuitive picture corresponding to the formal semantics is that at any given time the past and the present are completely determined while there are several alternative paths which the future may take. Given this semantics, we cannot in general say that a sentence of the form Fq is either true or false at a time t. Instead, we can only say that Fq is true at t from the perspective of some particular history which passes through t. Of course, if Fq is true at t for every history to which t belongs, then we have it that LFq is true at t regardless of the history we choose and hence Fq is true at t *simpliciter*. Thus, the settledness operator turns out to be a kind of truth operator within the formal language **TGL**, and if neither q nor $\neg$q is settled at t we say that there is a truth value gap for q at time t. Notice that if q contains no occurrences of F and if q is true at t for some history passing through t, then since time only begins to branch in the future q must be true at t for every history passing through t. So if q contains no occurrences of the future tense operator F, then q and Lq are equivalent. Only sentences containing occurrences of F can suffer a lack of truth-value.

While Thomason's intent is clearly to allow for truth-value gaps for contingent future tense sentences, it does not appear that he or Gupta wishes to say that all contingent future tense sentences lack truth value. It may be the case that LFq is true at t even when Fq is contingent. For example, Thomason and Gupta (1980) suggest that a sentence like

9.11 The local bus will not arrive at your place of business on time.

may be settled at some time t. Their view seems to be that the future may remain undetermined in some respects while being determined in others. This view seems at least plausible and I will not contest it. Model-theoretically, this assumption must be accommodat- ed by restricting the members of Ht to those histories which are 'lawful', that is to those histories all of which represent alternative fulfillments of the same set of physical laws. Otherwise it is difficult to see how any future contingent statement could be settled unless it contained some reference to the past of a sort which is lacking in 9.11. This in turn would mean that in a TG-model a time t could not belong to two histories in which different sets of physical laws were operative. We might, however, have two disjoint histories h and $h_1$ and a one-to-one function f from the times in h onto the times in $h_1$ which preserves the earlier-than relation $\leq$, and we might have two times $t \in h$ and $t_1 \in h_1$ such that for all times $t_2 \leq t$ and all sentences q, $<t_2,h> \in [q]$ if and only if $<f(t_2),h_1> \in [q]$. Then different laws might be operative in h and $h_1$ even though h and $h_1$ are factually indistinguishable at least through the times t and $t_1$. Perhaps this is a case in which we should say that the times in h are co-present with their corresponding times in $h_1$. Either this or some other device will be essential if we are to provide an adequate interpretation of the language **TGL**.

This theory encounters a problem when we turn our consideration from counterfactual conditionals to the more exotic counterlegal conditionals. A counterlegal conditional is one which proposes as its hypothesis a situation which could only obtain if some physical law were violated, e.g., 9.12 If the gravitational constant were to increase by 1% beginning now, people would suffer more frequent fractures unless they developed heavier bones.

I suggest that 9.12 is an example of a counterlegal conditional which is not only comprehensible but also true. Furthermore, the antecedent of 9.12 certainly does not require any change in past history. Given a Stalnaker semantics for conditionals, 9.12 should be true now for some possible history h to which now belongs just in case the consequent of 9.12 is true now in the history $h_1$ at which the antecedent is true which is most similar to h. Given familiar restrictions against changing the past gratuitously, h and $h_1$ should share the same past. But then something strange happens. If 'now' in 9.12 denotes the same time t in h as 'now' in 9.12 denotes in $h_1$, then $h \in Ht$ and $h_1 \in Ht$ even though h and $h_1$ are not subject to the same physical laws. Once we allow this, we can no longer have contingent future tense sentences which are settled because we have no physical laws common to all alternative futures to guarantee their truth.

One way to attack this problem would be to use the co-presence relation mentioned by Thomason and Gupta but deleted from the reformulation of their model theory which I have provided. We might maintain that the times referred to in h and in $h_1$ by the word 'now' in 9.12 are not the same time although they are co-present times. They are what David Lewis might call temporal counterparts. This brings into focus an interesting feature of the Thomason-Gupta analysis. Depending on what happens, tomorrow may be one time instead of another. Now we certainly say that there may be many different tomorrows, but I don't think we intend by this that tomorrow could be one of many different times. Instead, I think we mean that the one and only tomorrow might turn out one of many different ways. While the notion of alternative futures or of alternative histories is not counterintuitive, the idea that these alternatives are made up of different times is not common. Of course one might argue that two times at which different sentences are true must be different times, but then every time would seem to be distinct from itself since future tense sentences are in general true at a time only relative to a particular history passing through that time. If we used this sort of argument to try to justify the Thomason-Gupta theory, we would be forced to the conclusion that any two histories must be completely disjoint. Even if this conclusion is not accepted and we adopt a 'co-presence' analysis of counterlegals, we must still explain why a sentence like 9.12 should require us to consider a co-present now while an ordinary counterfactual containing now in its antecedent does not. It would be better to posit a single linear time (or perhaps a single space-time) and to consider different events which might fill it.

If we accept the Thomason-Gupta picture of alternative histories made up of alternative times, there may be another way of handling the problem without insisting that 'now' in 9.12 denotes different times in the two histories h and $h_1$. Instead of a co-presence relation, we could introduce into our model theory an accessibility relation R on the set of histories. This accessibility relation would have to be relativized to times, so that in fact R would be a function which assigned to each time t an equivalence relation R(t) on Ht. We would then use R to interpret sentences of the form Lq. We could, that is, replace 9.10 with

9.13 $<t,h> \epsilon$ [Lq] iff for all $h_1$ such that $hR(t)h_1$, $h_1 \epsilon$ [q].

By doing this, we can explain counterlegals without recourse to co-presence within the framework of a modified TG-semantics while at the same time allowing for the possibility of settled contingent future tense sentences. If we do this, we can no longer take sentences to be settled at a time t simpliciter, but only at a time t relative to some R(t) equivalence class of histories. This may not be a bad thing, but it is much weaker than the position taken in Thomason and Gupta (1980). While this repair of Thomason and Gupta is technically possible, I prefer a model theory which is not motivated by the view that there are not only alternate histories but also alternate times. Alternate times might be necessary to interpret conditionals whose antecedents require that time have a cyclical structure, etc., but for ordinary conditionals, including most counterlegal conditionals, such devices are not necessary. In the next section I will develop an alternative model theory for **TGL** which is formally equivalent to the modified TG-model theory presented here but which is not motivated by such assumptions about alternate times.

# 10 Pseudo-branching Time and Settledness

Without the assumption that there are alternate times which stand in no temporal relation to each other, we can produce much the same effect as that which results from the Thomason-Gupta semantics by letting possible worlds play a role similar to that of temporal branches or histories. As an alternative to TG-models, I suggest that we interpret the formal language **TGL** by means of ordered hextuples $<T,W,\ll,R,f,[]>$ satisfying the following conditions for all $t,t_1 \epsilon$ T, all $w,w_1 \epsilon$ W, and all sentences q and r of **TGL**:

10.1 – 10.11 are the same as 4.1 – 4.11.

10.12 – 10.14 are the same as 4.15 – 4.17.

10.15 For each $t \epsilon$ T let Ht = $\{<w,w_1>$: for all q and all $t_1$ such that $t_1 = t$ or $t_1 \ll t$, $<t_1,w> \epsilon$ [q] iff $<t_1,w_1> \epsilon$ [q]$\}$.

10.16 R is an equivalence relation on W.

10.17 $<t,w> \epsilon$ [Lq] iff for all $w_1$ such that $wRw_1$ and $<w,w_1> \epsilon$ Ht, $<t,w_1> \epsilon$ [q].

We see that two worlds w and $w_1$ share the same past up to time t just in case $<w,w_1> \epsilon$ Ht. Thus, Ht plays the same role in this semantics as it did in the theory of TG-models. As in the case of TG-models, Ht can be defined in terms of other items in our models and need not itself be an item in our models. Intuitively, the relation R tells us which worlds in W share the same physical laws. Then q is settled at time t in world w just in case q is true at time t in every world $w_1$ which shares the same physical laws as w and shares the same past with w up to time t. We notice that if q contains no occurrences of the future tense operator F, then $[q] = [Lq]$ just as in the case of TG-models. In fact, we can easily turn one of our present models into a modified TG-model of the sort discussed at the end of the last section. Let's define a relation GT on pairs of times and worlds such that $<t,w>GT<t_1,w_1>$ iff $t = t_1$ and $<w,w_1> \epsilon Ht$. Then let T+ be the set of GT-equivalence classes of time-world pairs. We will let $<t,w>$ be the GT-equivalence class of $<t,w>$. Next define a relation $\leq$ on T+ such that $<t,w> \leq <t_1,w_1>$ iff $t \ll t_1$ and $<w,w_1> \epsilon Ht$. Then $<T+,\leq,R,f,[]>$ closely resembles a modified TG-model since worlds play much the same role in our present models as do histories in TG-models. The primary difference in these derived models and TG-models is that f is a class-selection function rather than a Stalnakerian world-selection function. If we begin with a model $<T,W,\ll,R,f,[]>$ such that for any q, t, and w the set f(q,t,w) has at most one member, and if we let $s(q,t,w) = \phi$ if $f(q,t,w) = 0$ and let $s(q,t,w) \epsilon f(q,t,w)$ otherwise, then $<T+,\leq,R,s,[]>$ becomes a full-fledged TG-model with a settledness-accessibility relation R.

We note that there exist modified TG-models which are not equivalent to models of the sort just defined. This is because in a TG-model there may be a time t between two times $t_1$ and $t_2$ such that no time co-present with t is between two times co-present with $t_1$ and $t_2$ and related to each other by $\leq$.

One important difference between the model theory developed in this section and the original theory of TG-models is that we cannot speak simply of a sentence being true or false at a time. We must, in fact, speak of sentences being true or false at time-world pairs. But we can introduce the notion of truth-value gaps in somewhat the same way Thomason does. Once again we interpret our settledness operator L as a truth (or 'supertruth') predicate in our formal language. Where neither Lq nor ¬Lq is true at a time t and a world w, we say that there is a truth value gap at t and w for q. Just as in Thomason's original model theory, all purely past and present tense sentences have a truth value at each time-world pair, but future tense sentences may lack truth values at some time-world pairs. Interpreting **TGL** in this way, the past and present are completely determined while the future is in general only partially determined.

Something needs to be said about the role which the concept of an 'actual world' plays in the model theory developed in this section. I would distinguish the world from all possible worlds. By 'the world' I mean that which both you and I occupy, all that there is, or the totality of things. By 'possible world' I mean a way the world might have been or might be. I think of a possible world as a pattern of properties and relations together with a function which 'fits' concrete individuals into niches in this pattern. (For details, see Nute (1985).) The term 'actual world' would on my view denote the way the world actually is. Since the world is not the same thing as the way the world is, the terms 'the world' and 'the actual world' designate two different things. If we accept a model theory based on pseudo-branching time and we assume that the future is not completely determined, then there is no such thing as the way the world is. On this view, the world might yet be any number of different ways, none of which it is yet. If we accept this view, there is no 'actual world'; there is only the evolving world and the many different ways it might have been or might yet be.

We could distinguish, then, between all those merely possible worlds, those ways the world might have been but clearly can now never be, and those possible worlds each of which accurately describes the world to the extent it has so far been determined. These latter we might call 'actually possible' worlds. Where t is the present, the actually possible worlds will be some R-equivalence class of the set of worlds which accurately represent the world up to t. More exactly, the actual possible worlds comprise the R-equivalence class of these 'historically possible' worlds all the members of which are governed by the actual physical laws. We can say, then, that a sentence in **TGL** is true *simpliciter* iff it is true now in every actually possible world.

I believe that no great problems will arise if we add the tense operators H and G and the tensed conditional operators >PP>, >PF>, >FP> and >FF> to the language **TGL**. We can adapt the model theory of this section to the interpretation of these operators in a straightforward fashion. Since we are allowing truth value gaps we will not in general have $[Hq] = [\neg P \neg q]$ and $[Gq] = [\neg F \neg q]$ as we did when our models were based on linear time. I will not provide the details for such an expansion of our model theory.

# 11 Edelberg Inferences

There are two very interesting inference rules recorded by Thomason and Gupta (1980) involving the settledness operator L. These are:

Edelberg 1: From $L\neg q$ and $L(q > r)$ to infer $q > L(q \supset r)$.

Edelberg 2: From $L\neg q$, $q > Lq$ and $L(q > r)$ to infer $q > Lr$.

In a footnote, Thomason and Gupta mention stronger versions of these two inference principles:

Edelberg 3: From $L(q > r)$ to infer $q > L(q \supset r)$.

Edelberg 4: From $q > Lq$ and $L(q > r)$ to infer $q > Lr$.

These principles, all of which Thomason and Gupta endorse, motivate the second, more complicated model theory in their paper. Their goal was to develop a semantics which validated the Edelberg inferences but which also validated the Stalnakerian principle CEM. For the first model theory developed by Thomason and Gupta, which closely resembles the theory of TG-models developed in section 9 above, the Edelberg inferences can only be insured by imposing a restriction which seems both *ad hoc* and incorrect. Thus, we have the development of the more complicated, second model theory in Thomason and Gupta (1980).

As Thomason and Gupta observe, the restriction required to guarantee that the Edelberg inferences are validated by a class selection function semantics for conditionals is not so counterintuitive as the restriction required for a world selection function semantics. The only 'advantage' to be gained by using a world selection function semantics is that CEM turns out to be valid. Since I consider CEM to be a disadvantage rather than an advantage, I believe the extra complications of the second Thomason-Gupta model theory are unnecessary. All that is required, then, is to spell out the conditions for satisfying the Edelberg inferences in a class selection function semantics like that developed in section 10.

I suggest two further restrictions on our theory of conditionals in the context of pseudo–branching time. These two restrictions are more than strong enough to validate the Edelberg inferences. Both concern the notions of historical and physical possibility built into our model theory.

Consider a time-world pair $<t,w>$ and a sentence q. At which time-world pairs should we look in evaluating at $<t,w>$ a conditional with q as antecedent? We want all of those time-world pairs which are reasonably similar to $<t,w>$ at which q is true. Suppose we have another world $w_1$ such that w and $w_1$ have common physical laws and a common history up to at least time t. It is completely reasonable to think that any time-world pair at which q is true which is reasonably similar to $<t,w>$ is also reasonably similar to $<t,w_1>$. If w and $w_1$ both accurately describe the world up until now, we have no way of choosing between them since they only differ in their descriptions of the future which is yet to be determined. Any time-world pair reasonably similar to either should certainly be included in our actual deliberations. Thus I propose the following restriction for our model theory.

11.1 If $<w,w_1> \epsilon$ Ht and $wRw_1$, then $f(q,t,w) = f(q,t,w_1)$.

This restriction together with 10.12 gives us the following quite reasonable result.

11.2 If $<w,w_1> \epsilon$ Ht and $wRw_1$ and $w_1 \epsilon [q]$, then $<t,w_1> \epsilon f(q,t,w)$.

In fact, we should get an even stronger result which cannot be stated precisely. If w and $w_1$ share the same laws and the same history up to t, and if $t_1$ is reasonably similar to t (which will depend upon context and upon the particular antecedent q), then we will also want $<t_1,w_1>$ to be a member of f(q,t,w).

One interesting consequence of 11.2 is that we should expect the principle

CS: $(q \wedge r) \supset (q > r)$

to be invalid. Where r is a contingent sentence which depends on the future in a way that makes it indeterminate, we could certainly have a time t and two worlds w and $w_1$ which share the same laws and the same history up to t such that q is true at both $<t,w>$ and $<t,w_1>$ and r is true at $<t,w>$, but r is not true at $<t,w_1>$. Then given 11.2, $q \wedge r$ is true at $<t,w>$ but $q > r$ is not. Thus commitment to a theory of indeterminant time could provide additional reason to reject the principle CS, a principle which has already received considerable criticism. It should be noticed, though, that a modified version of CS,

CSL: $L(q \wedge r) \supset (q > r)$

escapes this particular criticism unscathed.

The second restriction I propose depends on the reasonableness of treating similarly worlds which share laws and histories, in much the same way as did the first restriction.

11.3 If $<t_1,w_1> \epsilon f(q,t,w)$, $<w_1,w2> \epsilon Ht$, $w_1Rw2$, and $<t_1,w2> \epsilon [q]$, then $<t_1,w2> \epsilon f(q,t,w)$.

The motivation for 11.3 should be clear. Again, we might endorse a stronger principle which can only be stated informally: if $<t_1,w_1> \epsilon f(q,t,w)$ and $t_2$ is reasonably close to t (given q and the context) and $w_1Rw2$ and either $<w_1,w2> \epsilon Ht_1$ or $<w_1,w2> \epsilon Ht_2$, then $<t_2,w2> \epsilon f(q,t,w)$. I feel much less confident of this principle than I do of the one corresponding to 11.2.

The restrictions 11.1 and 11.3 are sufficient to guarantee all four of the Edelberg inferences. 11.1 also guarantees the following very strong thesis:

11.4 $(q > r) \supset L(q > r)$.

If we add our tensed conditional operators $>PP>$, $>PF>$, $>FP>$ and $>FF>$ to the language **TGL**, we find that 11.1 is also strong enough to guarantee all of the theses produced by replacing the ordinary conditional operator in 11.4 by one of the tensed conditional operators. The only reason I can see for opposing 11.4 and its tensed counterparts is a commitment to CS, and such a commitment seems to me to be a mistake. 11.4 will certainly hold where q and r concern only the present and the past. Where q or r concern the future, we should surely want to say that an intensional conditional is only true if it is true regardless of the particular alternative future which is actualized. Finally, 11.1 and 11.3 together allow us to strengthen the Edelberg inferences in the following ways:

Edelberg 5: From $q > r$ to infer $q > L(q \supset r)$.

Edelberg 6: From $q > Lq$ and $q > r$ to infer $q > Lr$.

## 12 Loose Ends

We have explored a number of interesting issues involving the interaction of tense and conditionality, but much remains to be done. One important task which remains is the axiomatization of the logics characterized by the various model theories developed in this paper. Efforts in this direction will likely result in further refinement of the model theories themselves, and probably in alternative refinements which will compete for acceptance. Another avenue for further investigation, at which I have hinted repeatedly, is the adaptation of the suggestions in this paper to an interval semantics for time. Still another interesting problem which has been completely ignored in this paper is the analysis of conditionals involving progressive tenses. There is also a need to investigate the role which such temporal adverbs as 'since' and 'until' play in the truth conditions of conditionals in which they occur. I think that the progressive tenses always play an intensional role, and 'since' and 'until' play an intensional role in conditional contexts which they do not always play in other contexts. These additional intensional operators complicate the analysis of conditionals in ways which will only be untangled through considerable effort. Despite the large and growing literature in conditional logic, the problems of tense are only just beginning to attract the attention of conditional logicians. This paper, together with those by Thomason and Gupta and by van Fraassen, are only a beginning.

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